Name: ______________________ Section: ______________________

Right Sums

We have a couple of different ways to write down R_n.
We will generally choose the most appropriate tool for the type of problem we are working with.

1. Intuitively and Graphically: in Right Sums, the rectangle hits the curve at its **right** endpoint.

$$\begin{aligned} R_n &= (\text{height of rect 1})\Delta x + (\text{height of rect 2})\Delta x + \cdots + (\text{height of rect n})\Delta x \\ &= f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x \end{aligned}$$

2. Concisely: $R_n = \sum_{i=1}^{n} f(x_i)\Delta x$

3. When computing R_n, find $\Delta x = \dfrac{b-a}{n}$ and compute $x_1, x_2, \ldots, x_n$ where $x_i = a + i \cdot \Delta x$.

$$\begin{aligned} R_n &= f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x \\ &= \Delta x\Big(f(x_1) + f(x_2) + \cdots + f(x_n)\Big) \end{aligned}$$

Left Sums

We also have several ways to write L_n.

1. Intuitively and Graphically: in Left Sums, the rectangle hits the curve at its **left** endpoint.

$$\begin{aligned} L_n &= (\text{height of rect 1})\Delta x + (\text{height of rect 2})\Delta x + \cdots + (\text{height of rect n})\Delta x \\ &= f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x \end{aligned}$$

2. Concisely: $L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x$

3. When computing L_n, find $\Delta x = \dfrac{b-a}{n}$ and compute $x_0, x_1, \ldots, x_{n-1}$ where $x_i = a + i \cdot \Delta x$.

$$\begin{aligned} L_n &= f(x_0)\Delta x + f(x_1\Delta x + \cdots + f(x_{n-1})\Delta x \\ &= \Delta x\Big(f(x_0) + f(x_1) + \cdots + f(x_{n-1})\Big) \end{aligned}$$

The Midpoint Rule

In the midpoint rule, the rectangle hits the curve in the **middle**. The i^{th} interval is $[x_{i-1}, x_i]$. The midpoint of this interval is written $\bar{x}_i = \frac{x_{i-1}+x_i}{2}$.

1. Concisely: $M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$

2. When computing M_n, find $\Delta x = \dfrac{b-a}{n}$, find $x_0, x_1, \ldots, x_n$, and average them to find $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$.

$$\begin{aligned} M_n &= f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + \cdots + f(\bar{x}_n)\Delta x \\ &= \Delta x\Big(f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)\Big) \end{aligned}$$